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# Robust Synchronization of a Class of Robot Manipulators

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**Artículo de Investigación**

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**Abstract:** In this work, an adaptive control strategy for the synchronization of robotic manipulators is presented, demonstrated its stability, and verified with numerical results. The idea of synchronization is that various systems that may have completely different dynamics behave in a way that there exists no residual difference in their outputs. Here we present an approach for synchronizing robot manipulators despite unmodeled dynamics and parametric uncertainties, external disturbances, and parametric and structural differences of the robots. It is achieved with the help of a nonlinear controller with robust characteristics that only require the measurement of the angular positions. The uncertain functions are grouped into a new state that is, together with the other states of the system, estimated by a high-gain observer. With the estimated states, feedback is implemented based on linearization.

Finally, the proposed methodology is demonstrated for a two-degree-of-freedom (DOF) robot manipulator, and numerical results are presented.

**Keywords**: *Robot synchronization, Synchronization, Robust synchronization.*

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## Introduction

Synchronization is a phenomenon that has many examples in natural processes, such as the perfectly coincided oscillation of two pendulum clocks hanging from the same base [11], the synchronous firing of neurons [2],[5] or the symmetry of animal gaits [12]. As in these examples, interconnections in the systems achieve synchronization without external interference; We speak of self-synchronization. Additionally, we find numerous examples in different mechanical and electrical structures, such as transmitter-receiver systems, quadruped robot movements [3] etc. where the synchronization is achieved by external inputs and couplings, because of which we speak of controlled synchronization. This article focuses on the controlled synchronization of robot manipulators. We find many applications in production processes where robotic systems' synchronous behavior is necessary for producing equal-quality parts. In surgery, new minimal invasive robotic systems have been developed [8] that require the synchronization of the robot with the trajectory that the operating surgeon generates.

While the control of robot manipulators is a classical control problem, the problem of the synchronization of robots has not received much attention. We can find some approaches in [10] where the system's parameters are estimated by an observer using only angular positions. Using those estimates, an adaptive control strategy is realized. However, this technique requires the exact knowledge of the system's dynamics, which results in a non-robust approach. Therefore, in a realistic case, there is no knowledge of the friction's terms, parameter variations, etc.

This article assumes that the robot system's parameters and dynamics are uncertain and that only the angular positions can be measured. Departing from the ideas presented in [4] we use the proposed robust nonlinear control scheme for the Multiple Input Multiple Output (MIMO) case. The methodology achieves the synchronization of an arbitrary number of robots despite structural and parametric differences of the robots, and it is robust against external perturbations, friction, and parameter variations. After transforming the system into a linearizable canonical form, the uncertain dynamics and parameters are lumped into a new state. This new state is unknown, as well as the angular velocities; because of this, a high-gain observer estimates it. With the estimated states, a stabilizing controller is implemented based on linearization. Finally, the robots are connected in a mutual pattern that achieves the synchronization between the robots and concerns a trajectory given by the user.

## Materials and Methods

Let us consider a robotic manipulator that consists of $w$ links and has $m$ rotatory degrees of freedom that create the generalized angular positions $q_i$, $i = 1...m$. We assume that it is possible to generate $m$ torques $\tau_i$, $i = 1 ... m$ in the link connections, for example, with the help of electrical motors, hydraulic systems, etc. It was presumed that it is possible to measure the angular positions of links at each point in time while the availability of the angular velocity was not postulated. The robot's links were modeled as perfectly stiff, i.e., bending and vibration effects were neglected. With the help of the

Euler-Lagrange or similar equations, we can derive the following model of a robot with $m$ rotatory degrees of freedom:

$$\ddot{q} = M(q)^{-1}\left(\tau - C\left(q, \dot{q}\right)\dot{q} - g(q) - p\left(\dot{q}\right)\right)$$
Eq. 1

$M(q) \in R^{m\times m}$ is the symmetric, positive definite inertia matrix while $C\left(q, \dot{q}\right)\dot{q} \in R^{m}$ represent the Coriolis and centrifugal forces. $g(q) = \frac{\partial}{\partial q}E_{pot} \in R^{m}$ denotes the gravity forces, and the friction in the element connections is represented by the function $p\left(\dot{q}\right) \in R^{m}$. We decided to use the static friction model that proposed (Hellsen et al., 2000). The following equation represents it:

$$p_i\left(\dot{q}_i\right) = B_{v_i}\dot{q}_i + B_{f_{i,1}}\left(1 - \frac{2}{1+e^{2\omega_{i,1}\dot{q}_i}}\right) + B_{f_{i,2}}\left(1 - \frac{2}{1+e^{2\omega_{i,2}\dot{q}_i}}\right)$$
$$i = 1 \dots m$$
Eq. 2

$B_v$ is used to model the viscous friction, while the remaining terms approximate the Coulomb and Stribeck friction effects. We carry out the following transformation:

$$\left[x_1 \ \vdots \ x_m\right] = \left[q_1 \ \vdots \ q_m\right]$$
$$\left[x_{m+1} \ \vdots \ x_{2m}\right] = \left[\dot{q}_1 \ \vdots \ \dot{q}_m\right]$$
Eq. 3

Now (1) becomes a nonlinear $m \times m$ MIMO system, that is characterized by $n = 2m$ first order differential equations:

$$\dot{x} = f(x) + g_1(x)\tau_1 + \dots + g_m(x)\tau_m$$
$$y = \left[x_1 \ \dots \ x_m\right]^T$$
Eq. 4

With the states $x \in R^{n}$, the system input $\tau \in R^{m}$ and the system output $y \in R^{m}$. The system is characterized by the function $f(x) \in R^{n}$ and the matrix $g(x) \in R^{n\times n}$:

$$f(x) = \left[x_{m+1} \ \vdots \ x_{2m} \ M(x)^{-1}\left(- C\left(x, \dot{x}\right)\dot{x} - g(x) - p\left(\dot{x}\right)\right)\right.$$
$$g(x) = \left[0_{m\times m} \ 0_{m\times m} \ 0_{m\times m} \ M(x)^{-1}\right]$$
Eq. 5

For the synchronization of two or various robotic manipulators, we will presume that every system fulfills the following assumptions:

- A.1: Only the angular positions $\left[q_1 \dots q_m\right]$ can be measured at each point in time, i.e., not all the states $x_i, i = 1 \dots n$ of the system are available.
- A.2: There is no exact knowledge of the structure and the coefficients of $M(q)$, $C\left(q, \dot{q}\right)$, $g(q)$, and $p\left(\dot{q}\right)$.
- A.3: The robotic manipulators may be strictly different, but they all have the same degrees of freedom and the same inputs.

Numerous synchronization designs exist, such as serial or parallel master-slave models, etc. (Nijmeijer et al., 2003). However, this work will discuss the mutual synchronization pattern, where synchronous behavior is achieved with the interaction between the robots. The robots are arranged in a network, and every robot can be connected to all the other robots. Let us suppose we have a number of $l$ robots. For mutual synchronization the trajectories of reference $y_{ref_{i,k}}$ with $i = 1 \dots l$, $k = 1 \dots m$ of the robot $i$ for the degree of freedom $k$ are calculated as follows:

$$y_{ref_{i,k}} = y_{d_k} - \sum_{j=1, j\neq i}^{l} K_{cp_{i,j}}\left(y_{i,k} - y_{j,k}\right)$$
Eq. 6

Where $y_d \in R^{m}$ is the desired trajectory given by the user, which is equal for all the robots and has to be smooth. $K_{cp_{i,j}}$ are the so-called coupling factors. They define how strong the robot $i$ will interact with the robot $j$. High values of the coupling factors will lead to a fast synchronization between the robots, and low values will lead to a fast synchronization of the robots with the desired trajectory $y_d$.

The synchronization of all robot manipulators is achieved if $lim_{t\to\infty}|y_{ref_{i,k}}(t) - y_{i,k}(t)| \to 0$ for $i = 1 \dots l$ and $k = 1 \dots m$. It is straightforward that this is only possible if also $lim_{t\to\infty}|y_{d_k}(t) - y_{i,k}(t)| \to 0$ for $i = 1 \dots l$ and $k = 1 \dots m$. The synchronization problem can be formulated as designing the interconnections between the robots and creating control feedback for the robots. In the next chapter, we will propose a robust control feedback strategy well-suited for robot mutual synchronization.

## Results and Discussion

To implement the proposed feedback scheme, the system has to be transformed on Burnes Isidori Normal Form. Because this transformation requires the knowledge of the relative degree vector, we will use the following definition [7]:

Definition 2: (Relative Degree) The relative degree vector $\left[r_1 \dots r_m\right]$ of an affine MIMO system as in (4) is defined by:

1. $L_{g_j}L_f^{k}h_i(x) = 0$ for all $x$ close to $x_0$ and $1 \leq i, j \leq m, 0 \leq k \leq r_i - 2$
2. The matrix $A\left(x_0\right)$ is nonsingular

$$A\left(x_0\right) = \left[L_{g_1}L_f^{r_1-1}h_1\left(x_0\right) \ \cdots \ L_{g_m}L_f^{r_1-1}h_1\left(x_0\right) \ \vdots \ \ddots \ \vdots \ L_{g_1}L_f^{r_m-1}h_m\left(x_0\right)\right.$$

With this definition, we can find that, for the robot manipulators, $A(x) = M(x)^{-1}$ and that the relative degree of every input is $r_i = 2$. As $\zeta = r_1 + \dots + r_m = n$, the system has full order and, therefore, has no internal dynamics.
Now we can carry out the transformation $z = \phi(x)$, $\phi: R^n \rightarrow R^n$, with:

$$\phi(x) = \left[z_{1,1}\ z_{2,1}\ z_{1,2}\ \vdots\ z_{2,m}\ \right] = \left[h_1(x)\ L_f h_1(x)\ h_2(x)\ \vdots\ L_f\right.$$

Eq. 7

With this transformation the system (4) is linearizable and becomes:

$$\dot{z} = \left[\dot{z}_{1,1}\ \dot{z}_{2,1}\ \dot{z}_{1,2}\ \vdots\ \dot{z}_{2,m}\ \right] = \left[z_{2,1}\ \alpha_1(z) + \sum_{j=1}^{m} \beta_{j,1}(z)\tau_j\ z_{2,2}\right.$$

$$y = \left[y_1 \dots y_m\right]^T = \left[z_{1,1} \dots z_{1,m}\right]^T$$

Eq. 8

In the case of robot manipulators, the transformation $z = \phi(x)$, $\phi: R^n \rightarrow R^n$ is always a diffeomorphism and thereby $z = \phi(x)$ is an invertible transformation. This means that if we can control (8) we can also control (4). The vector $\alpha(z): R^{2m} \rightarrow R^m$ is defined by $\alpha_i(z) = L_f^2 h_i\left(\phi^{-1}(z)\right)$ and the matrix $\beta(z): R^{2m} \rightarrow R^{m\times m}$ by $\beta_{j,i}(z) = L_{g_j} L_f h_i\left(\phi^{-1}(z)\right)$. We find that $\alpha(z) = \left[f_{m+1}\left(\phi^{-1}(z)\right) \dots f_{2m}\left(\phi^{-1}(z)\right)\right]^T$ and $\beta(z) = M\left(\phi^{-1}(z)\right)^{-1}$. Thus, the linearizing controller $\tau = \beta(z)^{-1}(v - \alpha(z))$ is called the perfect control. If we choose $v_i$ for $i = 1 \dots m$ as follows

$$v_i = \dot{z}_{2,i} = \ddot{y}_i = \ddot{y}_{ref_i} - \rho_{1,i}\left(\dot{y}_i - \dot{y}_{ref_i}\right) - \rho_{2,i}\left(y_i - y_{ref_i}\right)$$

Eq. 9

the outputs of the system can follow any affine vector of trajectories of reference $y_{ref} \in C^2$ without any permanent error.

Remark 1: The controller $\tau = \beta(z)^{-1}(v - \alpha(z))$ requires the exact knowledge of all the states $z_i$ as well as the knowledge of $\alpha_i(z) = L_f^2 h_i(x)$ and $\beta_{j,i}(z) = L_{g_j} L_f h_i(x)$ for $i = 1 \dots m$, $j = 1 \dots m$ at each point in time.

However, as we have assumed in assumption A.2, in the case of the robot manipulators, we have no exact knowledge of the structure and the coefficients of $M(q)$, $C\left(q, \dot{q}\right)$, $g(q)$ and $p\left(\dot{q}\right)$ which means that also $\alpha(z)$ and $\beta(z)$ are uncertain. Besides, according to assumption A.1, only the angular positions $y = \left[z_{1,1} \dots z_{1,m}\right]^T = \left[q_1 \dots q_m\right]^T$ can be measured while the angular velocities $\dot{y} = \left[z_{2,1} \dots z_{2,m}\right]^T = \left[\dot{q}_1 \dots \dot{q}_m\right]^T$ are unknown.

Following the ideas presented in [4], [1]), and [9] where the controller requires only the least prior knowledge about the system (8) and can stabilize the system at the origin or make it follow any affine trajectory. The control scheme does not require the knowledge of $\alpha(z)$ and $\beta(z)$. The idea is to lump these uncertain terms into a new observable state that can be reconstructed from the available angular positions $\left[q_1 \dots q_m\right]$.
We introduce the new variable vector $\Theta \in R^m$, which contains the uncertain functions $\alpha(z)$ and $\beta(z)$ for $i = 1 \dots m$:

$$\Theta_i(z, \tau) = \alpha_i(z) + \sum_{j=1}^{m}\left(\beta_{j,i}(z) - \beta_{e_{j,i}}(z)\right)\tau_j$$

Eq. 10

$\beta_e(z) \in R^{m\times m}$ is a user-defined approximation of $\beta(z)$ that has to fulfill $sign\left(\beta_e(z)\right) = sign(\beta(z))$. With this we can rewrite the system (8), for $i = 1 \dots m$:

$$\dot{z}_{1,i} = z_{2,i}$$

$$\dot{z}_{2,i} = \Theta_i(z, \tau) + \sum_{j=1}^{m} \beta_{e_{j,i}}(z)\tau_j$$

Eq. 11

Now we augment our system by $m$ additional states $\eta_i(t) = \Theta_i(z, \tau)$ with $i = 1 \dots m$. In this way (11) becomes:

$$\dot{z}_{1,i} = z_{2,i}$$

$$\dot{z}_{2,i} = \eta_i(t) + \sum_{j=1}^{m} \beta_{e_{j,i}}(z)\tau_j$$

$$\dot{\eta}_i(t) = \Xi_i(z, \eta, \tau)$$

Eq. 12

Where $\Xi_i(z, \eta, \tau) = \frac{\partial\Theta_i}{\partial z}\frac{dz}{dt} + \frac{\partial\Theta_i}{\partial\tau}\frac{d\tau}{dt}$, in assumption A.1 we have supposed that we have no exact knowledge about all the states $x_i$. Consequently, the new state vector $\eta(t) \in R^m$ is also unknown. To solve this problem, we construct the following high-gain observer that is based on the available states $y = \left[z_{1,1} \dots z_{1,m}\right]$.

$$\dot{\hat{z}}_{1,i} = \hat{z}_{2,i} + L\kappa_{1,i}\left(z_{1,i} - \hat{z}_{1,i}\right)$$

$$\dot{\hat{z}}_{2,i} = \hat{\eta}_i + \sum_{j=1}^{m} \beta_{e_{j,i}}(z)\tau_j + L^2\kappa_{2,i}\left(z_{1,i} - \hat{z}_{1,i}\right)$$

$$\dot{\hat{\eta}}_i = L^3\kappa_{3,i}\left(z_{1,i} - \hat{z}_{1,i}\right) \quad i = 1 \dots m$$

Eq. 13

Now we have to choose the coefficients $\kappa_{i,j}$ in such a way that the polynomials $s^3 + \kappa_{1,i}s^2 + \kappa_{2,i}s + \kappa_{3,i}$, $i = 1 \dots m$ have poles with negative real parts. $L$ is a tuning parameter that strongly influences the error dynamics.
Based on the estimates of the uncertainties $\eta(t)$ and the estimates of $\left[z_{2,1} \dots z_{2,m}\right]^T$ we can construct the following linearizing-like feedback controller

$$\tau = \beta_e(z)^{-1}\left(v - \hat{\eta}\right)$$

Eq. 14

With the input vector $v \in R^m$ that is defined as:

$$v_i = \ddot{y}_{ref_i} - \rho_{1,i}\left(\hat{z}_{2,i} - \dot{y}_{ref_i}\right) - \rho_{2,i}\left(z_{1,i} - y_{ref_i}\right),$$
$$i = 1 \dots m$$

Eq. 15

Proposition 1: The robust feedback method consists of the dynamic estimator (13) and the linearizing controller (15), that was constructed using the estimates of $\Theta$ (i.e. $\eta(t)$) and $z$ that are provided by the high-gain observer.

Proof: The proof of stability is equal for all the $m$ degrees of freedom. Because of this, we will carry out a parallel proof for all the degrees of freedom, and $i = 1 \dots m$ will be valid. For the stability of the observer, we define an estimation error $e_i \in R^3$ in the following way: $e_{j,i} = L^{r-j+1}\left(z_{j,i} - \hat{z}_{j,i}\right)$, $j = 1, 2$ and $e_{3,i} = \eta_i - \hat{\eta}_i$. Now, using (12) and (13) we can write the error dynamics $\dot{e}_i$ as:

$$\dot{e}_{1,i} = L\left(-\kappa_{1,i}e_{1,i} + e_{2,i}\right)$$
$$\dot{e}_{2,i} = L\left(-\kappa_{2,i}e_{1,i} + e_{3,i}\right)$$
$$\dot{e}_{3,i} = -L\kappa_{r+1,i}e_{1,i} + \Xi_i$$

Eq. 16

Or written in Matrix form:

$$\dot{e}_i = L\left[-\kappa_{1,i}\ 1\ 0\ \ -\kappa_{2,i}\ 0\ 1\ \ -\kappa_{3,i}\ 0\ 0\right]e_i + \left[0\ 0\ \Xi_i\right]$$
$$= LA_i(\kappa)e_i + \Gamma_i$$

Eq. 17

The matrix $A_i(\kappa)$ is Hurwitz if the poles of the polynomial $s^3 + \kappa_{1,i}s^2 + \kappa_{2,i}s + \kappa_{3,i}$ are in the left pane of the complex plane. If this is the case, then, according to Lyapunov, there exists a positive definite and symmetric matrix $P_i$ such that $P_iA_i + A_i^TP_i = -I_n$ where $I_n$ is the identity matrix of dimension $n$. Now we choose $V_i\left(e_i\right) = e_i^TP_ie_i$ as Lyapunov function and get:

$$\dot{V}_i\left(e_i\right) = \frac{\partial V_i\left(e_i\right)}{\partial e_i}\dot{e}_i = -L|e_i|^2 + 2e_i^TP_i\Gamma_i$$
$$\leq -L|e_i|^2 + 2|P_i||e_i||\Gamma_i|$$

Eq. 18

If $\Gamma_i$ satisfies $|\Gamma_i| < r_1$ and $|e_i| < r_2$ for some $r_1 > 0$ and $r_2 > 0$ then $|P_i||e_i||\Gamma_i|$ is a bounded function. Let $\mu_i > 0$ be some positive constant and $2|P_i||e_i||\Gamma_i| < \mu_i$. We can write $\dot{V}_i\left(e_i\right) \leq -L|e_i|^2 + \mu_i$ for the stability of the observer $|e_i| \leq \sqrt{\frac{\mu_i}{L}}$ has to be fulfilled for all $i$. We can see that the estimation error $e_i$ depends directly on $L$. As $L$ increases, $e_i$ and the estimation error bound will decrease. Because of this, $L$ should be chosen as big as possible.

We conclude: As all $\Gamma_i$ are bounded, if $L > L^* > 0$ then $e(t) \rightarrow 0$ for $t \rightarrow \infty$ and $\left(\hat{z}, \hat{\eta}\right) \rightarrow (z, \eta)$. With this we conclude, that (13) and (14) yield asymptotical stabilization of the system (8).

To illustrate the proposed control scheme, we will now apply the methodology to the case of a robot manipulator with $m = 2$ rotatory degrees of freedom. With the help of the Euler-Lagrange or similar equations we can calculate $M(q)$, $C\left(q, \dot{q}\right)$, $g(q)$ of (1) as follows:

$$M_{11} = m_1l_{c1}^2 + m_2l_1^2 + m_2l_{c2}^2 + I_1 + I_2 + 2m_2l_1l_{c2}cos\left(q_2\right)$$
$$M_{12} = m_2l_{c2}^2 + m_2l_1l_{c2}cos\left(q_2\right) + I_2$$
$$M_{21} = m_2l_{c2}^2 + m_2l_1l_{c2}cos\left(q_2\right) + I_2$$
$$M_{22} = m_2l_{c2}^2 + I_2$$
$$C_{11} = -m_2l_1l_{c2}sin\left(q_2\right)\dot{q}_2$$
$$C_{12} = -m_2l_1l_{c2}sin\left(q_2\right)\left(\dot{q}_1 + \dot{q}_2\right)$$
$$C_{21} = m_2l_1l_{c2}\dot{q}_1sin\left(q_2\right)$$
$$C_{22} = 0$$
$$g\_\{1\}(q) = g\ sin(q_1\left(m_1l_{c1} + m_2l_1\right) + m_2gsin\left(q_1 + q_2\right)l_{c2}$$
$$g_2(q) = m_2gl_{c2}sin\left(q_1 + q_2\right)$$

We will use the same friction term $p\left(\dot{q}\right) \in R^2$ as in (2). Again $q \in R^2$ are the angular positions of the links while $\dot{q} \in R^2$ are the angular velocities and $\tau \in R^2$ are the torques that are applied to the links. $l_1$, $l_2$ are the lengths of the links and $l_{c1}$, $l_{c2}$ are the distances to their centers of mass. $m_1$, $m_2$ are the masses of the two elements, $I_1$, $I_2$ are their moments of inertia (including the motors, joints, etc.) and $g$ is the acceleration of gravity.

After replacing $\left[x_1, x_2\right]^T = \left[q_1, q_2\right]^T$, $\left[x_3, x_4\right]^T = \left[\dot{q}_1, \dot{q}_2\right]^T$ and $M^*(x) = M^{-1}(x)$ we can rewrite our system (1):

$$\left[\dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \dot{x}_4\ \right] = \left[x_3\ x_4\ f_3(x) + M^*_{1,1}\tau_1 + M^*_{1,2}\tau_2\ f_4(x) + M^*_{2,1}\tau_1 + M^*_{2,2}\right.$$

Eq. 19

Where

$$\left[f_3(x), f_4(x)\right]^T = M(x)^*\left(-C\left(x, \dot{x}\right)\dot{x} - g(x) - p\left(\dot{x}\right)\right).$$

Using Definition 2, we find that the relative degree is $r = 2$. Now we introduce the augmented state vector $\eta(t) = \left[\eta_1(t), \eta_2(t)\right]^T$ and the user-defined approximation of $\beta_e(z) of \beta(z) = M\left(\phi^{-1}(z)\right)^{-1}$ and get:

$$\dot{z}_{1,1} = z_{2,1}$$
$$\dot{z}_{2,1} = \eta_1(t) + \beta_{e_{1,1}}(z)\tau_1 + \beta_{e_{2,1}}(z)\tau_2$$

$$\dot{\eta}_1(t) = \Xi_1(z, \eta, \tau)$$
$$\dot{z}_{1,2} = z_{2,2}$$
$$\dot{z}_{2,2} = \eta_2(t) + \beta_{e_{1,2}}(z)\tau_1 + \beta_{e_{2,2}}(z)\tau_2$$
$$\dot{\eta}_2(t) = \Xi_2(z, \eta, \tau)$$

Eq. 20

For the reconstruction of the angular velocities $\left[z_{2,1}, z_{2,2}\right]^T$ and the extended state $\eta(t)$ we construct the high-gain observer:

$$\dot{\hat{z}}_{1,1} = \hat{z}_{2,1} + L\kappa_{1,1}\left(z_{1,1} - \hat{z}_{1,1}\right)$$
$$\dot{\hat{z}}_{2,1} = \hat{\eta}_1 + \sum_{j=1}^{2} \beta_{e_{j,1}}(z)\tau_j + L^2\kappa_{2,1}\left(z_{1,1} - \hat{z}_{1,1}\right)$$
$$\dot{\hat{\eta}}_1 = L^3\kappa_{3,1}\left(z_{1,1} - \hat{z}_{1,1}\right)$$
$$\dot{\hat{z}}_{1,2} = \hat{z}_{2,2} + L\kappa_{1,2}\left(z_{1,2} - \hat{z}_{1,2}\right)$$
$$\dot{\hat{z}}_{2,2} = \hat{\eta}_2 + \sum_{j=1}^{2} \beta_{e_{j,2}}(z)\tau_j + L^2\kappa_{2,m}\left(z_{1,2} - \hat{z}_{1,2}\right)$$
$$\dot{\hat{\eta}}_2 = L^3\kappa_{3,2}\left(z_{1,2} - \hat{z}_{1,2}\right)$$

Eq. 21

With the estimates of (21) we can implement the following controller:

$$\left[\tau_1\ \tau_2\right] = \left[\beta_{e_{1,1}}(z)\ \beta_{e_{1,2}}(z)\ \beta_{e_{2,1}}(z)\ \beta_{e_{2,2}}(z)\right]^{-1}\left[v_1 - \hat{\eta}_1\ v_2 - \hat{\eta}\right.$$

Eq. 22

In order to follow the smooth trajectory of reference $y_{ref} = \left[y_{ref_1}, y_{ref_2}\right]^T$, we choose $v = \left[v_1, v_2\right]^T$ as:

$$v_1 = \ddot{y}_{ref_1} - \rho_{1,1}\left(\hat{z}_{2,1} - \dot{y}_{ref_1}\right) - \rho_{2,1}\left(z_{1,1} - y_{ref_1}\right)$$
$$v_2 = \ddot{y}_{ref_2} - \rho_{1,2}\left(\hat{z}_{2,2} - \dot{y}_{ref_2}\right) - \rho_{2,2}\left(z_{1,2} - y_{ref_2}\right)$$

Eq. 23

Now the coupling factors were chosen as $K_{cp} = 10$ while the we consider arbitrary initial conditions for $q_{1i}, q_{2i}$ and $\dot{q}_{1i}, \dot{q}_{2i}$. The controller was switched on after 5 seconds, and after 10 seconds, a perturbation torque $\tau_{pert} = 10$Nm was applied to both link connections of all the robots. $\tau_{pert}$ was turned off after 15 seconds. We chose an arbitrary smooth function for the trajectory $y_d \in R^2$. The following variables were chosen equally for all four robots:

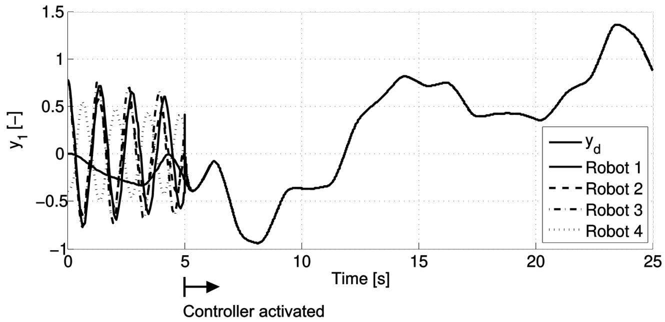
Table 1. Parameters of the systems.

| $L$ | $g$ | $l_1$ | $l_2$ | $l_{c1}$ | $l_{c2}$ | $I_1$ | $I_2$ |
|---|---|---|---|---|---|---|---|
| 20 | 9.81 | 0.35 | 0.3 | 0.175 | 0.145 | 0.0064 | 0.004 |

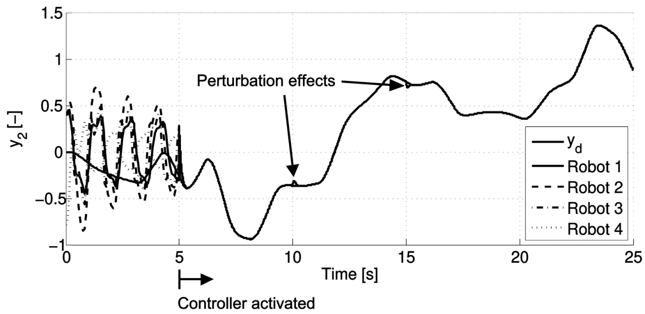
In the table $L$ is the high-gain parameter, $l$ is given in meters and $I$ is in $Kg.m^2$.

In Fig. 1, we can see the free oscillation of the uncontrolled robots in the first 5 seconds. After this period, the robots follow the arbitrary, user-given trajectory $y_d$ with very small errors. In Fig. 2, one can identify the torques generated to compensate for the perturbation.

**Figure 1**. (a) $y_{d_1}$ and system output $y_1$. (b) $y_{d_2}$ and system output $y_2$.



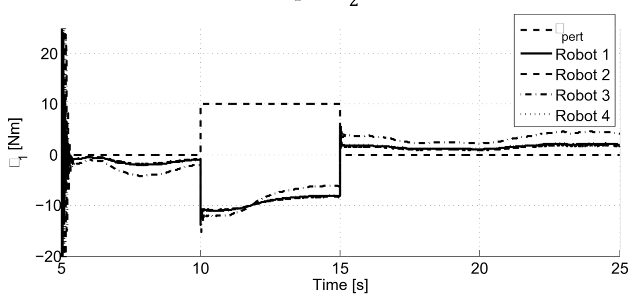

a)



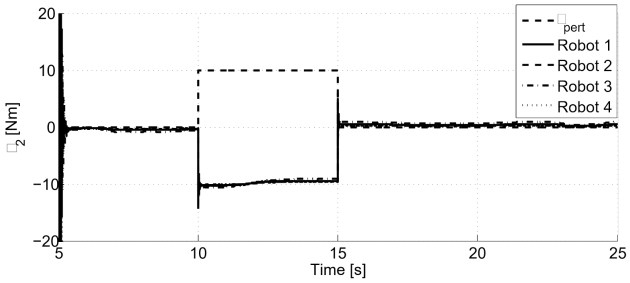

b)

Indeed, note that in Fig. 1, the effect of the perturbation is very low (after 5 and after 10 seconds). This shows that the approach does not require the estimation of the perturbation.

Figure 2. (a) $\tau_{pert_1}$ and system input $\tau_1$. (b) $\tau_{pert_2}$ and system input $\tau_2$.




a)




b)

## Conclusions

In this work, we have presented a robust control scheme that achieves synchronization of robot manipulators with an arbitrary number of degrees of freedom. It compensates unmodeled dynamics, uncertain or time-varying parameters, and external perturbations and requires only the measurement of the angular positions at each point in time. The central feature of this approach is that the uncertainties are lumped into an extended state, which a high-gain observer reconstructs. Based on this estimation, a linearizing-like control law is implemented that achieves the synchronization in combination with a mutual connection pattern of the robots. The methodology was demonstrated for the case of a 2 DOF robot manipulator and validated by numerical results. The proposed control scheme can also be applied to other mechanical systems, such as robot manipulators with linear degrees of freedom and in combination with other synchronization patterns.

## Conflicts of Interests

The authors solemnly declare that we are not and shall not be in any situation which could give rise to a conflict of interest.

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