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Multiple Signal Transmission Using Chaos Synchronization

Transmisión De Señales Múltiples Usando Sincronización De Caos

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Resumen: En este trabajo, se expone una forma segura de transmisión de varias señales basada en la sincronización del caos. La idea es transmitir de un modo seguro tantas señales como la cantidad de estados del sistema caótico lo permita. Se requiere que el atractor caótico transmitido no cambie para mantener el nivel de seguridad.
El método se basa en la teoría de control de múltiples entradas y múltiples salidas (MIMO) para sistemas no lineales. Además, el Transmisor y el Receptor se sincronizan a pesar de las incertidumbres en ambos sistemas. En este sentido, el esquema de comunicación segura es robusto. Se ilustra el resultado utilizando la sincronización de sistemas similares
Palabras claves: Sincronización de Caos, Transmisión Segura Robusta,Control No Lineal.*a*

Abstract: In this contribution, we present the secure transmission of several signals based on chaos synchronization. The idea is to transmit as many signals as the system states in a secure manner. It is required that the transmitted chaotic attractor must not change to maintain the security level.
The method is based on the Multiple-Input and Multiple-Output (MIMO) control theory for nonlinear systems. Moreover, the Transmitter and Receiver synchronize in spite of the uncertainties in both systems. In this sense, the secure communication scheme is robust. We illustrate the result using the synchronization of similar systems.
Keywords: Chaos Synchronization, Robust Secure Transmission, Nonlinear Control.

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Introduction

In the last two decades, the synchronization problem of chaotic systems has been widely studied, see, for instance, [1], [2], and [3]. The reasons for the interest in this problem are the potential applications and the synchronization phenomena. The main application is secure communication, where it is proved that relevant information can be transmitted and recovered using chaotic synchronization, and it has been demonstrated that the method is feasible, see for example, [4] and [5]. The other main interest is the dynamical phenomena of synchronization, where some relevant efforts have been made [6], [7], from where some open problems persist. For instance, one problem is the synchronization of strictly different systems, and the other is the synchronization of dynamical systems with different order [8]. The former deal with the problem of synchronize two systems whose models come from two different phenomena. The latter concerns the synchronization of dynamical systems that evolves in different spaces, in other words, a system with nonequal dimension. Synchronization of strictly different systems is still a problem under investigation.

Secure communication is understood as the transmission of signals or messages masked or hidden into the chaotic attractor of the transmitter system. Then the Receiver provides the transmitted signal or message via a particular unmasking algorithm. In this manner a simplex communication scheme is constructed; however, until the authors' knowledge, the secure transmission schemes consist in transmitting a single message or signal provided that the signal does not destroy the chaotic behavior of the transmitter. Several chaos-based communication schemes have been published. Among these, two basic configurations can be identified. (i) An approach consists of adding the signal to the chaotic carrier, which is transmitted to the receiver. That is, the master system comprises the full-state model, whereas the slave system is composed of a reduced model, and the transmitted state completes the receiver system. (ii) Another transmitter/receiver design is based on the full state model of the driving and response systems [9]. That is, both drive and response systems are represented by dynamical systems of the same order. The homogeneous-synchronization configuration has been recently addressed via parameter modulation. Kocarev and Parlitz have proposed a generalization of these approaches, which extends the capabilities for constructing synchronized systems. Their approach enables the message signal to be integrated as a driving signal. However, the message signal can be recovered only under ideal conditions [10].

In this report, we illustrate that multiple signals can be transmitted using a single transmitter system and a single receiver using nonlinear Multiple-Input and Multiple-Output control techniques. In this way, a stereo audio signal can be transmitted, or different conversations can be embedded to be

transmitted at the same time. Moreover, chaotic behavior should be maintained.
The control strategy is based on a diffeomorphic transformation obtained from the derivatives of the output functions along the system vector fields for a single signal see [5]. Then as the transmitter system poses many inputs as transmitted signals, it is required the same number of control inputs in the receiver in order to recover each signal.

The writing is organized as follows: Section 2, presents the secure communication based on robust synchronization of similar chaotic systems, in Section 3 an application for secure communication is illustrated and finally Section 4 presents the conclusions of the work.

Materials and Methods

The key procedure consists in synchronize chaotic systems with same order and model. To this end, we consider chaotic systems in the following form

$$\begin{aligned} \dot{x_T} &= f(x) + \sum_{i=1}^{n} \Gamma_{i,T}(x) s(t)_i \\ y_T &= h_i(x) \\ \dot{x_R} &= f(x) + \sum_{i=1}^{n} \Gamma_{i,R}(x) u_i \\ y_R &= h_i(x) \end{aligned} \qquad \text{Ec. 1}$$

where vector fields $f(x): \Omega \to R^n$ are sufficiently smooth and similar, with $\Omega \subset R^n$ being the attraction region, $\Gamma_{i,T}(x)$, $\Gamma_{i,R}(x)$ are the input vectors with $\Gamma_{i,T}$ being unknown and $y_T, y_R \in R^n$, $h_{i,T}, h_{i,R}: R^n \to R$ are the real-valued output functions, $s_i(t)$ are the signals to be transmitted and the signals u_i stand for the control signals to achieve synchronization, and once the systems are synchronous, the recovered messages are reproduced by the control command signals. These considerations define a Multiple-Input and Multiple-output system.
Therefore, we use the nonlinear control theory for MIMO systems. To begin with, from Master and Slave systems, the synchronization error system is given as

$$\begin{aligned} \dot{x_e} &= \Psi(x_e) + \Sigma_{i=1}^{n} \left(\Gamma_{i,T}(x_e) s_i(t) - \Gamma_{i,S}(x_e) u_i\right) \\ y_{i,e} &= h_{e,i} \end{aligned} \qquad \text{Ec. 2}$$

where $x_e \in R^n$ is the state vector of the synchronization error system, $\Psi(x_e) = f(x_T) - f(x_R)$, $y_{i,e} = h_{i,e} = x_{i,T} - x_{i,R}$. The underlying idea of the nonlinear feedback control is to find functions u_i such that the desired dynamical behavior is induced for any initial condition $x_{e,0} = x_e(0)$ in an attraction basin $U_o \subset R^n$. Then, from the Lie-based geometric approach, it is possible to find a transformation $z = \Phi(x_e), \Phi: R^n \to R^n$, for $z \in \Omega \subset U$, such that the affine form of the synchronization error system takes a linearizable canonical form. We depart from the relative degree definition for a MIMO system [11].

Definition 1. The Multiple Input and Multiple Output affine system (2) has the relative degree vector $(\rho_1, \rho_2, \dots, \rho_n)$ at the point x_e^0 if:

i) $L_{\Gamma_{j,R}} L_\Psi^k h_{i,e}(x_e) = 0$, for all $1 \le j, i \le n$, $k < \rho_i - 1$, and for all x_e in the neighborhood of x_e^0
ii) the $n \times n$ matrix

$$A_{x_e} = \begin{pmatrix} L_{\Gamma_{1,R}} L_\Psi^{\rho_1 - 1} h_{1,e} & \cdots & L_{\Gamma_{n,R}} L_\Psi^{\rho_1 - 1} h_{1,e} \\ L_{\Gamma_{1,R}} L_\Psi^{\rho_2 - 1} h_{2,e} & \cdots & L_{\Gamma_{n,R}} L_\Psi^{\rho_2 - 1} h_{2,e} \\ \vdots & \ddots & \vdots \\ L_{\Gamma_{1,R}} L_\Psi^{\rho_n - 1} h_{n,e} & \cdots & L_{\Gamma_{n,R}} L_\Psi^{\rho_n - 1} h_{n,e} \end{pmatrix} \qquad \text{Ec. 3}$$

is nonsingular at $x_e = x_e^0$.

Remark 1. The previous definition of the relative degree is for a system with the same number of inputs signals as output signals. Therefore, the relative degree matrix A_{x_e} is square.

Suppose that the system (2) gives $\rho_i = 1$, then the matrix A_{x_e} is given as follows

$$A_{x_e} = \begin{pmatrix} L_{\Gamma_{1,R}} h_{1,e} & 0 & \cdots & 0 \\ 0 & L_{\Gamma_{2,R}} h_{2,e} & \cdots & 0 \\ \vdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & L_{\Gamma_{n,R}} h_{n,e} \end{pmatrix} Ec.4$$

Due to the invertibility of the matrix A_{x_e}, a diffeomorphic transformation can be determined. Such a transformation is given by $z = \Phi(x_e) = \left[h_{1,e}, h_{2,e}, \dots, h_{n,e}\right]^T$, since the relative degree $\rho_i = 1$ the linearizable system is given by

$$\begin{aligned} \dot{z_i} &= \zeta_i(z) + \vartheta_i(z) u_i \\ y_{i,e} &= z_i,\ i = 1,2, \dots, n \end{aligned} \qquad \text{Ec. 5}$$

where $\zeta_i(z) = L_\Psi h_{i,e}(\Phi(z)^{-1})$ and $\vartheta_i(z) = L_{\Gamma_{i,R}} h_{i,e}(\Phi(z)^{-1})$. Thus, from (5) the linearizing controllers are given by

$$u_i = \frac{1}{\vartheta_i(z)}(-\zeta_i(z) + \nu_i) \qquad \text{Ec. 6}$$

Where $i = 1,2, \dots, n$ and $\nu_i = K_i z_i$ is the new control that leads the system state to a prescribed reference, in this case, it leads the trajectories of the system (2) to the origin.

Remark 2. The controller (6) is the so-called perfect control since it requires the perfect knowledge of the functions $\zeta_i(z) = L_\Psi h_{i,e}(\Phi(z)^{-1})$ and $\vartheta_i(z) = L_{\Gamma_{i,R}} h_{i,e}(\Phi(z)^{-1})$, in order to accomplish the synchronization.

At this point, we can assume the following:
Assumption 1. Only the state vector x_e is available for feedback.
Assumption 2. $\vartheta_i(z)$ is an injective function, bounded away from zero.
Assumption 3. There exists a scalar function $\widehat{\vartheta}_i(z)$ available for feedback such that $sign[\widehat{\vartheta}_i(z)] = sign[\vartheta_i(z)]$ at any $(z) \in U^0 \subset R^n$ of (z^0).

Thus, for a realistic case, we consider that these functions are unknown, in seek of clarity $L_{\Gamma_{i,R}} h_i(\Phi(z)^{-1})$ represents the way as the control input is affecting the system and $L_\Psi h_i(\Phi(z)^{-1})$ represents the dynamics that should be compensated (uncertain terms and parameters as well as the transmitted signals). These unknown functions represent a source of uncertainty due to parameters and non-modeled dynamics. Moreover, if there exist external perturbations or systems faults, they appear in function $L_\Psi h_i(\Phi(z)^{-1})$ or $L_{\Gamma_{i,R}} h_i(\Phi(z)^{-1})$. For these reasons, the controllers (6) are not physically realizable. To overcome this problem, we propose a representation that lumps the uncertain terms into a new uncontrollable but observable state variable. Following ideas reported in [12], [13] and [14], the system (5) can be written in an extended form by defining $\delta_i(z) = \vartheta_i(z) - \widehat{\vartheta}_\iota(z)$, $\Theta_i(z,u) = \zeta_i(z) + \delta_i(z)u_i$, note that $\Theta_i(z,u_i)$ lumps the uncertainties. Now, let us define $\eta_i = \eta_i(t) \equiv \Theta_i(z,u_i)$. Thus, the system (5) takes the form

$$\begin{aligned} \dot{z}_\iota &= \eta_i + \widehat{\vartheta}_\iota(z)u_i \\ \dot{\eta}_\iota &= \Xi_i(z,\eta_i,u_i,\dot{u}_\iota) \\ y_{i,e} &= z_i, \quad i = 1,2,\dots,n \end{aligned} \qquad \text{Ec. 7}$$

where $\Xi_i(\cdot) = \sum_{k=1}^{n}\left[\frac{\partial\Theta_k(\cdot)}{\partial z_k}\left(\eta_k + \widehat{\vartheta_k}(z)u_k\right)\right] + \delta_i(z)\dot{u}_\iota$, the augmented state η_i provides the dynamics of the uncertain function $\Theta_i(z,u_i)$, therefore, the system (7) is dynamically equivalent to system (5). Thus the ith linearizing-like control law is modified in the following form

$$u_i = \frac{1}{\widehat{\vartheta}_\iota(z)}(-\eta_i + \nu_i) \qquad \text{Ec. 8}$$

function $\widehat{\vartheta}_\iota(z)$ is an estimate of the function $\vartheta_i(z)$ and can be used provided that $Sign\left(\widehat{\vartheta}_\iota(z)\right) = Sign(\vartheta_i(z))$ [14]. Notice that the sign should be constant since any change of sign in function $\hat{\vartheta}$ implies a zero crossing, and the matrix A_{x_e} losses invertibility. Then under the linearizing control feedback (8), the states (z_i,η_i) converge to zero. Since the feedback controller yield bounded trajectories of state z, thus η_i is also bounded for all $t \geq 0$. More precisely, if z_i converges to an equilibrium point, then η_i also converges to an equilibrium point. Therefore, under feedback (8), the states of system (7) converge asymptotically to zero.

It is important to stress that the controller (8) is not realizable since the new state η_i is unknown. Therefore, a state estimator is proposed to estimate the uncertain states, which is given as follows

$$\begin{aligned} \dot{\hat{z}}_\iota &= \widehat{\eta}_\iota + \widehat{\vartheta}_\iota(z)u_i + \lambda_i\kappa_1(z_i - \widehat{z}_\iota) \\ \dot{\widehat{\eta}}_\iota &= \lambda_i^2\kappa_2(z_i - \widehat{z}_\iota) \quad i = 1,2\cdots,n \end{aligned} \qquad \text{Ec. 9}$$

where λ_i and κ_i are tuning parameters chosen in such way that the estimated values $(\widehat{z}_\iota,\widehat{\eta}_\iota) \rightarrow (z_i,\eta_i)$.

Proposition 1. Let $e \in R^2$ be an estimation error vector whose components are defined as: $e_1 = \lambda(z_i - \widehat{z}_\iota)$ and $e_2 = \eta_i - \widehat{\eta}_\iota$. For sufficiently large λ_i, the dynamics of the estimation error decays globally exponentially to zero if $Sign\left(\widehat{\vartheta}_\iota(z)\right) = Sign(\vartheta_i(z))$.

Proof: Combining systems (7) and (9) the dynamics of the estimation error can be written as follows

$$\dot{e} = \lambda_i De + \Psi(z,\eta_i,u_i) \qquad \text{Ec. 10}$$

where $\Psi(z,\eta_i,u_i) = [0,\Xi(z,\eta_i,u_i)]^T$ and the matrix D is

$$D = \begin{pmatrix} -\kappa_1 & 1 \\ -r\kappa_2 & 0 \end{pmatrix} \qquad \text{Ec. 11}$$

in which $r = \vartheta_i(z)/\widehat{\vartheta}_\iota(z)$ and $r > 0$ which implies $Sign\left(\widehat{\vartheta}_\iota(z)\right) = Sign(\vartheta_i(z))$ thus D is obviously Hurwitz. There exists a positive definite and symmetric matrix P such that $PD + D^TP = I_2$ where I_2 is the identity matrix of dimension 2. Choosing $V(e) = e^TPe$ as a the Lyapunov like function candidate, one has

$$\begin{aligned} \dot{V}(e) &= -\lambda_i|e|^2 + 2e^TP\Psi(z,\eta_i,u_i) \\ \dot{V}(e) &\leq -\lambda_i|e|^2 + 2|P||e||\Psi(z,\eta_i,u_i)| \end{aligned} \qquad \text{Ec. 12}$$

Let e be the estimation error. The function $\Psi(z,\eta_i,u_i)$ satisfies $|\Psi(z,\eta_i,u_i)| \leq r_1$ and $e \leq r_2$ for some $r_1 > 0$ and $r_2 > 0$, respectively. Thus, $|P||e||\Psi(z,\eta_i,u_i)|$ is a certain bounded function. Moreover, let $|P||e||\Psi(z,\eta_i,u_i)| \leq \epsilon$ be satisfied for some positive constant $\epsilon > 0$. In this way, we have

$$\dot{V}(e) \leq -\lambda_i|e|^2 + 2\epsilon \qquad \text{Ec. 13}$$

so that e tends to a set bounded by $|e| \leq \sqrt{2\epsilon/\lambda_i}$. The dependence on the estimation error e on λ_i deserves special attention. Note that as λ_i increases, e will decrease, which also decreases the exponential estimation error bound. This argument shows that with the proposed method, λ_i should be made as large as possible and this achieves the proof.

The previous proposition states that there exists a Lyapunov function that depends on parameter λ_i and provides the global exponential convergence of the estimation error dynamics. With the estimates of the states, the linearizing control low can be modified as follows

$$u_i = \frac{1}{\widehat{\vartheta}_\iota(z)}(-\widehat{\eta}_\iota + \nu_i) \qquad \text{Ec. 14}$$

An important characteristic is that this feedback only uses estimated values of the uncertain terms, which gives the feature of robust feedback, moreover it only requires the measure of the output states.

Thus, the Robust Control Scheme for hyperchaotic synchronization can be achieved by means of the state estimator (9) and the robust feedback (14). In this sense, it is required that the state estimator converges to the corresponding states. This result can be stated in the following proposition.

Proposition 2. Consider two chaotic/hyperchaotic systems with a synchronization error given by the system (2). The system (2) converges asymptotically to zero under the feedback (14) via the stabilization of the extended system (7).

Proof: Substituting the robust feedback (14) into (7) and considering the estimation error system (10), the closed-loop dynamics is given by

$$\begin{aligned} \dot{z}_i &= \eta_i + \hat{\vartheta}_i(z)u_i \\ \dot{\eta}_i &= \Xi_i(z, \eta_i, u_i, \dot{u}_i) \\ \dot{e} &= \lambda De + \Psi(z, \eta_i, u_i) i = 1,2,\cdots,n \end{aligned} \quad \text{Ec. 15}$$

Since $\eta_i = \Theta(z, u_i)$ and $u_i = \frac{1}{\hat{\vartheta}_i(z)}(-\hat{\eta}_i + \nu_i)$ it follows that $\eta_i = \mathfrak{Z}_i(z, \eta_i, e_i, u_i, t)$ (which can be computed from the first integral of $\dot{\eta}_i = \Xi_i(z, \eta_i, u_i, \dot{u}_i)$, i.e., $\eta_i = \int \Xi_i(z, \eta_i, e_i, u, \tau)d\tau$. Then, according to the Contraction Mapping Theorem, the state η_i can be expressed globally and uniquely as a function of the coordinates (z_i, e_i). Now, note that since the matrix D is Hurwitz by construction, and the nonlinear function $\Psi(z, \eta_i, u_i)$ is bounded, the estimation error system (15) is asymptotically stable. In this sense, given a compact set of initial conditions $\chi_i \subset R^2$ containing the origin, there exists an upper bound $u_{i,max}$ such that $u_i \leq u_{i,max}$ and a high gain estimator parameter λ_i such that χ_i is contained into the attraction basin. Hence, the closed-loop system is semi-globally practically stable, i.e., $(e_i, \eta_i) \to (0,0)$, and this achieves the proof.

Therefore, with these results, one can design controllers that achieve synchronization in spite of uncertainties. Thus, we apply these results to construct a secure communication system based on synchronization.

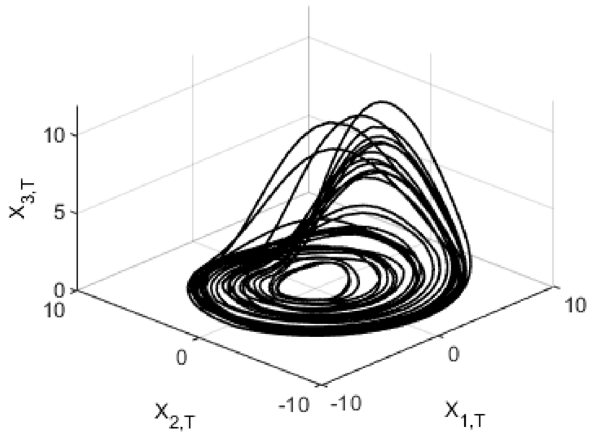
Results and Discussion

To illustrate our proposal, we consider two chaotic Rössler systems given by

$$\begin{aligned} \dot{x_1} &= -x_2 - x_3 \\ \dot{x_2} &= x_1 + ax_2 \\ \dot{x_3} &= (x_1 - d)x_3 + b \\ y_i &= x_i \end{aligned} \quad \text{Ec. 16}$$

one for the Transmitter and the other for the Receiver. For this case, the parameters were considered equal in both systems. Then we desire to send three signals without excessive deformation or destroy the chaotic Transmitter attractor. In this example we transmit a square periodic signal and two sinusoids, all with different amplitudes and frequencies. Once the signals have been injected into the Transmitter system, the attractor of the Transmitter is illustrated in Figure 1, where there is a slight deformation of the attractor, and it still provides a chaotic behavior.

Figure 1. Chaotic attractor for the Rössler Transmitter system when the transmitted signals are injected.



Therefore, the synchronization error system obtained from the Transmitter and Receiver system is given by

$$\begin{aligned} \dot{x}_{1,e} &= -x_{2,e} - x_{3,e} + s_1(t) - u_1 \\ \dot{x}_{2,e} &= x_{1,e} + a_R x_{2,e} + s_2(t) - u_2 \\ \dot{x}_{3,e} &= -(x_{1,e} - d_R)x_{3,e} + \delta(x_T, x_e) + s_3(t) - u_3 \\ y_{i,e} &= h_{i,e}, \qquad i = 1,2,3. \end{aligned} \quad \text{Ec. 17}$$

where the function $\delta(x_T, x_e)$ stands for the terms given by the nonlinear operation between systems. As was stated in the previous section, system (17) is a nonlinear Multiple-Input and Multiple-Output one, moreover the relative degree matrix $A_{x_e} = I_{3\times 3}$ which is invertible and satisfies the propositions of the previous section. Following the corresponding procedure, one has that the diffeomorphic transformation is given by $\Phi(x_e) = [h_{1,e}, h_{2,e}, h_{3,e}]^T$. Once we have the transformation, the system for the synchronization error can be written as

$$\begin{aligned} \dot{z_1} &= \zeta_1(z) - \vartheta_1(z)u_1 \\ \dot{z_2} &= \zeta_2(z) - \vartheta_2(z)u_2 \\ \dot{z_3} &= \zeta_3(z) - \vartheta_3(z)u_3 \\ y_i &= z_i, \qquad i = 1,2,3. \end{aligned} \quad \text{Ec. 18}$$

where the function $\zeta_i(z)$ comprises the uncertain terms since it represents the derivatives of the ith output along the vector field of the synchronization error system and the function $\vartheta_i(z)$ stands for the function of the input control and is considered uncertain, but by Assumption 3 there exists an estimated value given by $\hat{\vartheta}_i(z)$. Therefore, we can calculate the controllers as follows

$$u_i = \frac{1}{\hat{\vartheta}_i(z)}(-\zeta(z)_i + \nu_i) \quad \text{Ec. 19}$$

where the new dynamic is given by $\nu_i = k_i z_i$, however as was discussed above, the function $\zeta_i(z)$ is uncertain and the previous controller is not realizable. The idea is to lump the uncertain terms into a new state. Thus, the transformed system can be extended as follows

$$
\begin{aligned}
\dot{z}_1 &= \eta_1 - \widehat{\vartheta_1}(z)u_1 \\
\dot{\eta}_1 &= \Xi_1(\eta_1, z, u_1) \\
\dot{z}_2 &= \eta_2 - \widehat{\vartheta_2}(z)u_2 \\
\dot{\eta}_2 &= \Xi_2(\eta_2, z, u_2) \\
\dot{z}_3 &= \eta_3 - \widehat{\vartheta_3}(z)u_3 \\
\dot{\eta}_3 &= \Xi_3(\eta_3, z, u_3) \\
y_i &= z_i,\ i = 1,2,3.
\end{aligned}
\qquad \text{Ec. 20}
$$

This new system does not have the uncertainties of the synchronization error system; however, it has new states that provide the dynamic of the uncertainties. Again, the new states are not available from the system but can be estimated to have an estimated value, and the controllers can counteract them.

The state estimators for system (20) are given by

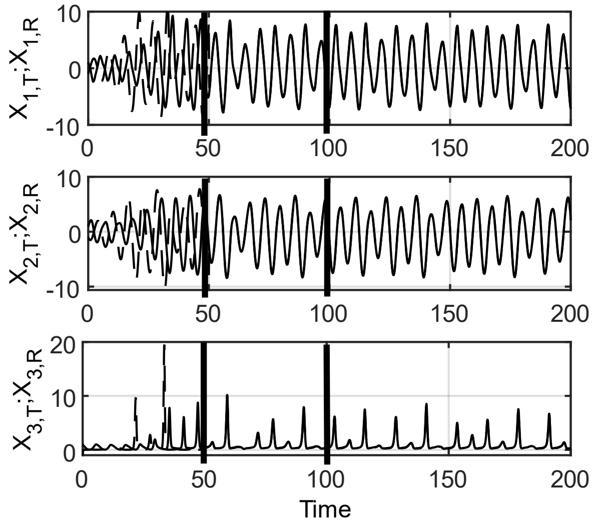
$$
\begin{aligned}
\dot{\hat{z}}_1 &= \widehat{\eta_1} - \widehat{\vartheta_1}(z)u_1 + L_1\kappa_{1,1}(z_1 - \widehat{z_1}) \\
\dot{\hat{\eta}}_1 &= L_1^2\kappa_{1,2}(z_1 - \widehat{z_1}) \\
\dot{\hat{z}}_2 &= \widehat{\eta_2} - \widehat{\vartheta_2}(z)u_2 + L_2\kappa_{2,1}(z_2 - \widehat{z_2}) \\
\dot{\hat{\eta}}_2 &= L_2^2\kappa_{2,2}(z_2 - \widehat{z_2}) \\
\dot{\hat{z}}_3 &= \widehat{\eta_3} - \widehat{\vartheta_3}(z)u_3 + L_3\kappa_{3,1}(z_3 - \widehat{z_3}) \\
\dot{\hat{\eta}}_3 &= L_3^2\kappa_{3,2}(z_3 - \widehat{z_3})
\end{aligned}
\qquad \text{Ec. 21}
$$

where this estimator provides the corresponding estimated values of the uncertain states η_i. With these estimates, we have that the controllers are given by

$$u_i = \frac{1}{\widehat{\vartheta_i}(z)}(-\widehat{\eta_i} + \nu_i) \qquad \text{Ec. 22}$$

Therefore, it is possible to determine the estimators and controller gains such that the closed-loop systems be stable, and applying these controllers to the corresponding systems, we obtain that the synchronization is achieved as illustrated in Figure 2, where the controllers were activated at $t \geq 50$ sec., after that the signals were applied or transmitted for $t_s \geq 100$ sec. Thus, it is observed that neither the chaotic attractor nor the synchronization is affected considerably when the signals are injected.

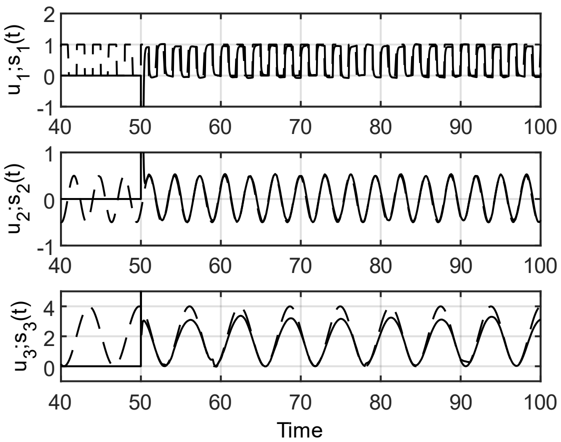
Figure 2. Synchronization of the Transmitter and Receiver systems, at $\mathrm{t} \geq 50$ sec. the synchronization is initiated, whereas for $\mathrm{t_s} \geq 100$ sec. the signals are injected into the Transmitter system.



Once the chaotic synchronization and the signals have been injected, the recovered transmitted signals are presented in Figure 3.

Note that the recovered signals are very close to the transmitter. Therefore, digital and analogous signals can be transmitted simultaneously in a secure way in spite of uncertainties in the Transmitter and Receiver systems.

Figure 3. Transmitted and recovered signals.



Conclusions

In this contribution, we presented that using chaos synchronization, multiple signals can be transmitted in a secure way. The proposal is such that analog and digital signals can be transmitted without deforming or destroying the chaotic behavior of the Transmitter and Receiver. Also, we considered the case when the controllers do not pose full information about the controlled system, in this way, the communication scheme is robust against unknown parameter values or external (signal transmitted) perturbations. The scheme is general for n order systems, therefore, in principle, it can be used to transmit n signals. The key assumption was that the systems are similar and the parameter variations were not considered, but it is being an understudy.

Conflicts of Interest

The authors solemnly declare that we are not and shall not be in any situation which could give rise to a conflict of interest.

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